instant of "landing" of the last grain of sand). When $k \to 1$, any subsequent motion is absent.

It is important to note that all conclusions about the existence of self-similar modes in the ejection of a medium by gas and, as a consequence, the power law of similarity expressed by $E_0 \sim h^\alpha$ remain valid for auger-hole blasting, even when ρ / ρ_0 and the effective value of χ behind the wave front are not constants, but functions of relative strain. This follows from dimensional considerations. A similar model is valid, for example, for approximately defining an explosion in a strongly fissured rock taking into account its gradual transformation into detritus. We would add that the existence of self-similar modes with expansion of a small cavity does not necessarily require the presence of a compression jump. An example of this is the expansion of a bubble in an incompressible fluid (the second stage in the Rayleigh problem).

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ON THE PROBLEM OF GLIDING OF A PLATE ON THE SURFACE OF A HEAVY IDEAL LIQUID OF FINITE DEPTH

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The two-dimensional problem of the flow around an arbitrary contour floating on the surface of a heavy ideal fluid of finite depth is considered. By using the results of [1 - 3] the problem mentioned is reduced by operational calculus methods in Sect. 1 to the determination of the pressure on the contour from an integral equation of the first kind with nonregular difference kernel of complex structure dependent on two dimensionless parameters λ and δ.

The case of gliding of an inclined plate is studied in detail in Sects. 2–4. An asymp-

totic solution of the integral equation obtained in Sect. 1 is given in Sect. 2 on the basis of results in [4], for large values of the dimensionless parameter λ and arbitrary $\delta \neq 1$. A solution of the integral equation is presented in Sect. 3 for small values of λ and $\delta < 1$ by the method proposed in [5]. Finally, for small values of λ, the case of $\delta > 1$, is considered in Sect. 4. This latter is singular in the sense that the Fourier transform of the kernel of the integral equation has two symmetrically located poles on the real axis. Integral equations with such and more general kernels have been investigated in [6], in which an asymptotic solution is given for the integral equation for small values of the parameter λ and a foundation is given for the method applied therein. However, the formulas presented therein are too complicated for practical application. An approximate solution is presented herein for a given particular form of the kernel of the integral equation, which is convenient for numerical computations.

Examples are considered in Sect. 5; graphs illustrating the efficiency of the proposed formulas for the whole range of variation of the parameters $\lambda \in (0, \infty)$ and $\delta \neq 1$ are cited.

1. Let us consider the two-dimensional problem of the flow around an arbitrary contour floating on the surface of a heavy ideal fluid of finite depth (Fig. 1).

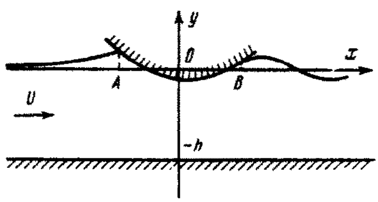

Fig. 1

As is known [1, 2], in the linear formulation we arrive at the following mixed boundary value problem:

$$\Delta\Psi_1(x', y') = 0, \quad \frac{\partial \Psi_1(x', -h)}{\partial x'} = 0 \tag{1.1}$$

$$p_1 - p_0 = -\rho g \eta_1(x') - \rho U \frac{\partial \Psi_1(x', y')}{\partial y'} = 0$$

$$\text{for } y' = 0, \ x' \in \Omega$$

$$\Psi_1(x', y') + U\eta_1(x') = 0 \qquad \text{for } y' = 0, \ x' \in \Omega$$

$$\Psi_1(x', y') + Uf_1(x') = 0 \qquad \text{for } y' = 0, \ x' \in \Omega'$$

Here $\Psi_1(x', y')$ is the perturbed motion stream function, $\eta_1(x')$ the equation of the free surface, $f_1(x')$ the equation of the streamlined contour, p_1 the hydrodynamic pressure, p_0 the atmospheric pressure, U the stream velocity, ρ the fluid density, g gravity.

It will henceforth be convenient to locate the origin at the middle of the projection of the wetted portion of the contour on the unperturbed stream surface and to pass to the dimensionless variables

$$(x', y') = l(x, y), \quad \Psi_1(x', y') = Ul\Psi(x, y) \tag{1.2}$$

$$p_1 - p_0 = \rho U^2 p, \quad [\eta_1(x'), f_1(x')] = l[\eta(x), f(x)]$$

Here $l = AB$ is half the length of the wetted zone of the plate (Fig. 1).

The relations (1.1) can be reduced in the abstract quantities (1.2) to

$$\Delta\Psi(x, y) = 0, \quad \frac{\partial\Psi(x, y)}{\partial x} = 0 \quad \text{for } y = -\frac{h}{l}$$

$$p = \frac{\Psi(x, y)}{\lambda} - \frac{\partial\Psi(x, y)}{\partial y} = 0, \quad \text{for } y = 0, \ |x| > 1 \tag{1.3}$$

$$\Psi(x, 0) = -f(x) \quad \text{for } |x| \leqslant 1, \quad \lambda = U^2(gl)^{-1}$$

By using the Fourier transformation the formulated problem can be reduced to the determination of the pressure in the wetting zone $p(x)$ from the following integral equation:

$$A \sin\frac{cx}{\lambda} + B\cos\frac{cx}{\lambda} + \int_{-1}^{1} p(u)\,du \int_0^\infty \frac{\cos\tau(x-u)\lambda^{-1}}{\tau \operatorname{cth}\delta\tau - 1} d\tau = \pi f(x) \tag{1.4}$$

Here c is a positive root of the equation $\tau \operatorname{cth} \delta\tau = 1$, which always exists for $\delta > 1$, A and B are arbitrary constants and $\delta = ghU^{-2}$.

The term outside the integral in the left side of (1.4) exists only for $\delta > 1$ and is the nonzero form of the free fluid surface in the corresponding homogeneous problem when the pressure on the whole surface is zero. In this case the arbitrary constants A and B can be selected for the problem under consideration [3] so that the perturbation of the free surface would vanish upstream ($x \to -\infty$).

It is not difficult to show that for this it is sufficient to set

$$A = -\pi a \int_{-1}^{1} p(u) \cos \frac{uc}{\lambda} du, \qquad B = \pi a \int_{-1}^{1} p(u) \sin \frac{uc}{\lambda} du \tag{1.5}$$

$$a = \lim_{\tau \to c} \frac{\tau - c}{\tau \operatorname{cth} \delta\tau - 1} = \frac{c}{1 - \delta(1 - c^2)}$$

after which we arrive at the integral equation to determine the pressure

$$\int_{-1}^{1} p(u) \left[\int_{0}^{\infty} \frac{\cos \tau (x - u) \lambda^{-1}}{\tau \operatorname{cth} \delta\tau - 1} d\tau - \pi a \sin \frac{c(x - u)}{\lambda} \right] du = \pi f(x), \quad |x| \leqslant 1$$

where the inner integral is here understood in the principal value sense. The integral equation obtained can be written in another form which will be convenient later

$$\int_{-1}^{1} p(u) N\left(\frac{x - u}{\lambda}\right) du = \pi f(x), \qquad |x| \leqslant 1 \tag{1.6}$$

$$N(t) = \int_{0}^{\infty} \left(\frac{1}{\tau \operatorname{cth} \delta\tau - 1} - \frac{2ac}{\tau^2 - c^2} \right) \cos \tau t d\tau - \pi a (\sin ct + \sin c |t|)$$

The integral

$$\int_{0}^{\infty} \frac{\cos \tau t}{\tau^2 - c^2} d\tau = -\frac{\pi}{2c} \sin |t|$$

has been used to obtain this form.

To construct effective asymptotic solutions of the integral equation (1.6) for large and small values of the parameter λ, let us apply the methods proposed in [4, 5] but paying special attention to those changes which are connected with the singularities of the integral equation obtained.

2. Let us consider the case of large λ. Following [4], let us expand the kernel of the integral equation (1.6) in series for large λ

$$N(t) = -\ln |t| + a_{30} + a_{31} t + a_{32} t^2 + a_{20} |t| + a_{23} |t|^3 + a_{33} t^3 + \\ + a_{12} t^2 \ln |t| + O(t^4 \ln |t|), \qquad t = (x - u) \lambda^{-1} \tag{2.1}$$

$$a_{30} = \int_{0}^{\infty} \left(\frac{1}{\tau \operatorname{cth} \delta\tau - 1} - \frac{1 - e^{-\tau}}{\tau} - \frac{2ac}{\tau^2 - c^2} \right) d\tau, \qquad a_{31} = -\pi ac$$

$$a_{32} = -\frac{3}{4} + \frac{1}{2} \int_{0}^{\infty} \left[1 + \tau - 2ac + \frac{1 - e^{-\tau}}{\tau} - \tau^2 \left(\frac{1}{\tau \operatorname{cth} \delta\tau - 1} - \frac{2ac}{\tau^2 - c^2} \right] \right) d\tau$$

$$a_{20} = -\pi / 2, \quad a_{12} = 0.5, \quad a_{23} = \pi / 12, \quad a_{33} = \pi ac^3 / 6$$

The expansion (2.1) of the kernel of the integral equation (1.6) differs from the expansion of the kernel examined in [4] by the presence of the odd members $a_{31}t$ and $a_{33}t^3$, which exert no substantial influence on the construction of the solution, but result only in additional calculations which need not be considered here. For the case we finally obtain

$$\omega(x) = p(x)\sqrt{1-x^2}, \qquad \omega(x) = \sum_{i=0}^{3}\sum_{j=0}^{[i/2]} \omega_{ij}(x)\lambda^{-i}\ln^j\lambda$$

$$\omega_{00} = \pi^{-1}P + \beta x, \quad \omega_{10} = \pi^{-1}P\,[4a_{20}\pi^{-2}S_1(x) - a_{31}x] + 2\beta a_{20}\pi^{-2}\,[2x - \Lambda(x)]$$

$$\omega_{20} = \pi^{-1}P\,\{(0.8069a_{12} + a_{32})(1-2x^2) + 32a_{20}^2\pi^{-4}\,[S_2(x) - 0.1508] -$$
$$-2\pi^{-2}a_{20}a_{31}\,[2x - \Lambda(x)]\} + \beta\,\{a_{12}x^3 + (a_{32} - 0.1931a_{12})\,x - 16a_{20}^2\pi^{-4}S_7(x)\}$$

$$\omega_{21} = -P\pi^{-1}a_{12}(1-2x^2) - a_{12}\beta x$$

$$\omega_{31}' = \pi^{-1}P\,[a_{12}a_{31}x - 2a_{12}a_{20}\pi^{-2}S_4(x)] - 2\beta\pi^{-2}a_{20}a_{12}\,[14x/3 - \Lambda(x)]$$

$$\omega_{30} = \pi^{-1}P\,[0.8889\pi^{-2}a_{12}a_{20}S_3(x) + 8a_{22}(3\pi^2)^{-1} + 64\pi^{-6}a_{20}^3S_5(x) + \tag{2.2}$$
$$+\pi^{-2}\,[6a_{22}(1+2x^2) - 0.19a_{20}^3]\,S_1(x) + \pi^{-2}\,[9a_{22} + 2(a_{32} + 0.8069a_{20}a_{12})]\,S_4(x) -$$
$$-a_{31}a_{32}x + 16\pi^{-4}a_{31}a_{20}^2S_7(x) - a_{31}a_{12}(x^3 - 0.1931x) - 3a_{33}x^3\} +$$
$$+\beta\,\{a_{12}a_{20}\pi^{-2}S_8(x) + 2a_{32}a_{20}\pi^{-2}\,[14x/3 - \Lambda(x)] +$$
$$+3a_{33}(x^2 - 0.5) - 16a_{20}^3\pi^{-6}S_9(x) + 2a_{22}\pi^{-2}S_{10}(x)\}, \quad \omega_{ij}(x) = \omega_{ij}$$

where

$$S_7(x) = -2x\,(0.8125 - 0.1067x^2 - 0.06x^4) + (0.7067 - 0.1467x^2 -$$
$$-0.06x^4)\,\Lambda(x), \quad S_8(x) = -0,025x + 2.667x^3 - 4xS_1(x) - (2.280 + \tag{2.3}$$
$$+1.333x^2)\,\Lambda(x), \quad S_9(x) = -9,309x + 1.860x^3 + 0.4354x^5 +$$
$$+(3.893 - 1.092x^2 - 0.2177x^4)\,\Lambda(x), \quad S_{10}(x) = 2.667x + 2x^3 -$$
$$-6xS_1(x) - (2 + x^2)\,\Lambda(x), \quad \Lambda(x) = (1-x^2)\ln(1-x)(1+x)^{-1}$$

and the functions $S_i(x)$, $i = 1, 2, \ldots, 5$ are presented and tabulated in [4].

The constant P in (2.2) has the meaning of lift of the plate and is determined from the condition of satisfying the obtained solution of the initial integral equation (1.6). The condition has the form

$$\pi^{-1}P\psi_1(\lambda) + \beta\psi_2(\lambda) = \gamma \tag{2.4}$$
$$\psi_1(\lambda) = a_{30} + 0.8106a_{20}\lambda^{-1} + (a_{32} + a_{12}' - 0.03287a_{20}^2 + 0.5a_{31}^2)\lambda^{-2} +$$
$$+(1.442a_{22} - 0.2702a_{12}a_{20} - 0.1807a_{20}a_{32} - 0.0245a_{20}^3 + 0.2702a_{31}^2a_{20})\,\lambda^{-3} +$$
$$+\ln 2\lambda\,(1 - a_{12}\lambda^{-2} + 0.1801a_{12}a_{20}\lambda^{-3}) + O(\lambda^{-4}\ln^2\lambda)$$
$$\psi_2(\lambda) = -0.5a_{31}\lambda^{-1} - 0.2702a_{20}a_{31}\lambda^{-2} - (1.125a_{33} + 0.2784a_{12}a_{31} + 0.5a_{32}a_{31} +$$
$$+0.1279a_{20}^2a_{31})\,\lambda^{-3} + 0.5a_{12}a_{31}\lambda^{-3}\ln\lambda + O(\lambda^{-4}\ln^2\lambda)$$

Formulas (2.2) and (2.3) yield a solution unbounded at the endpoints $x = \pm 1$. If boundedness of the pressure is required at the point $x = 1$ (from physical considerations, there should be no singularity at the plate trailing edge), then we arrive at the following relationship:

$$\pi^{-1}P\psi_3(\lambda) + \beta\,\psi_4(\lambda) = 0 \tag{2.5}$$
$$\psi_3(\lambda) = 1 - (0.4053a_{20} + a_{31})\,\lambda^{-1} + [a_{12} + (a_{12}a_{31} + 0.1351a_{12}a_{20})\,\lambda^{-1}]\times$$
$$\times\lambda^{-2}\ln\lambda - (a_{32} + 0.8069a_{12} + 0.04954\,a_{20}^2 + 0.4053a_{20}a_{31})\,\lambda^{-2} -$$
$$-(3a_{33} + 0.8069a_{12}a_{31} + a_{31}a_{32} + 0.2122a_{31}a_{20}^2 + 2.161a_{22} + 0.01895a_{12}a_{20} +$$
$$+0.1351a_{32}a_{20} + 0.01a_{20}^3)\,\lambda^{-3} + O(\lambda^{-4}\ln^2\lambda)$$
$$\psi_4(\lambda) = 1 + 0.4053a_{20}\lambda^{-1} + (a_{32} + 0.8069a_{12} + 0.2122a_{20}^2)\,\lambda^{-2} + (1.5a_{33} +$$
$$+2.161a_{22} + 0.1167a_{20}^3 + 0.9456a_{20}a_{32} + 0.6729a_{12}a_{20})\,\lambda^{-3} - (a_{12} +$$
$$+0.9456a_{20}a_{12}\lambda^{-1})\,\lambda^{-2}\ln\lambda + O(\lambda^{-4}\ln^2\lambda)$$

Let us determine the moment of the pressure relative to the origin

$$M = P\psi_2(\lambda) + \beta\psi_5(\lambda) \tag{2.6}$$

$$\psi_5(\lambda) = 1.571 + 0.8488a_{20}\lambda^{-1} + (0.8748a_{12} + 1.571a_{32} + 0.4017a_{20}^2)\lambda^{-2} + (2.716a_{22} + 1.698a_{20}a_{32} + 0.9231a_{12}a_{20} + 0.2752a_{20}^3)\lambda^{-3} - (1.571a_{12} + 1.698a_{12}a_{20}\lambda^{-1})\lambda^{-2}\ln\lambda + O(\lambda^{-4}\ln^2\lambda)$$

The plate equilibrium conditions, according to which the quantity P can be considered known and equal to the weight of the plate, must be added to these relations, and

$$M = P(1 - \lambda l_1 g U^{-2}) \tag{2.7}$$

where l_1 is the coordinate of the center of gravity of the plate relative to the trailing edge, which can also be considered known.

Conditions (2.4), (2.5) permit finding the unknown quantities β and γ which govern the location of the plate, as well as the length of the zone of contact λ which, as is easily shown, satisfies the equation

$$\lambda_1 = l_1 g U^{-2} = \lambda^{-1}[1 - \psi_2(\lambda) + \psi_3(\lambda)\psi_5(\lambda)(\pi\psi_4(\lambda))^{-1}] \tag{2.8}$$

As is seen, the length of the wetted zone is independent of the plate weight and is determined completely by the position of the center of gravity. Let us still find the total drag of the plate in an ideal fluid [1]

$$W = \beta P = -P^2\psi_3(\lambda)(\pi\psi_4(\lambda))^{-1} \tag{2.9}$$

Let us note that the drag is proportional to the square of the plate weight and diminishes as the center of gravity recedes from the trailing edge.

3. Let us consider the case of small λ. For small values of the parameter λ let us seek the approximate solution of the integral equation (1.6) by utilizing the method in [5]. To do this we take the pressure $p(x)$ in the form

$$p(x) = p_1\left(\frac{1+x}{\lambda}\right) + p_2\left(\frac{1-x}{\lambda}\right) - q(x) \tag{3.1}$$

where $p_i(t)$, $i = 1, 2$ are the solutions of the Wiener-Hopf integral equations

$$\int_0^\infty p_j(u) N_j(t-u)\,du = \pi\lambda^{-1} f_j(t), \quad j = 1, 2; \quad 0 \leqslant t < \infty \tag{3.2}$$

$$N_j(t) = \int_0^\infty \left(\frac{1}{\tau \operatorname{cth}\delta\tau - 1} - \frac{2ac}{\tau^2 - c^2}\right)\cos\tau t\,d\tau - \pi a \sin c|t| + (-1)^j \pi a \sin c|t|$$

$$f_j(t) = \gamma + (-1)^j \beta(1 - \lambda t) \tag{3.3}$$

and $q(x)$ is the degenerate solution of the integral equation as $\lambda \to 0$.

As has been shown in [7], and later in [8], the solution selected in the form (3.1)-(3.3) yields the zero term of the asymptotics of the solution of the integral equation (1.6) as $\lambda \to 0$ and $\delta < 1$. The next terms are exponentially decreasing of the form $\exp(-\varkappa\lambda^{-1})$, $\varkappa > \varepsilon > 0$.

The results of [5] can be utilized in the case $\delta < 1$ for the solution of the integral equation (1.6) in which we can set $a = 0$, hence we arrive at the final formulas

$$q(x) = (A\lambda)^{-1}(\gamma + \beta x), \quad p_i(t) = (A\lambda)^{-1}\gamma(\operatorname{erf}\sqrt{Bt} + e^{-Bt}\sqrt{A/\pi t}) - (-1)^i\beta(A\lambda)^{-1}[(\lambda t - 1)\operatorname{erf}\sqrt{Bt} + e^{-Bt}(\pi Bt)^{-1/2}(\lambda t - \sqrt{AB} + 0.5\lambda\sqrt{A/B} - \lambda A)] \quad (i = 1, 2) \tag{3.4}$$

An approximation of the form

$$(\tau^2 + C)^{-1} \sqrt{\tau^2 + B^2}, \qquad A = BC^{-1} = \delta\,(1 - \delta)^{-1}$$

was used to obtain these formulas for the Fourier transform of the kernel of the integral equation (1, 6).

Let us note that a more complex approximation was proposed in [9], which would reflect the character of the Fourier transform of the kernel of the integral equation (1, 6) at infinity more completely, and would yield a more exact result for the pressure distribution. Only the integral characteristics are investigated below, hence the simplest approximation is utilized which will supply campletely satisfactory results for the examples considered herein.

Requiring boundedness of the pressure at the point $x = 1$, we arrive at the condition

$$\gamma = -\beta z_1(\lambda), \qquad z_1(\lambda) = 1 + \lambda\,[C^{-1} - (2B)^{-1}] \tag{3.5}$$

Proceding from (3, 4), we obtain the lift of the plate and the moment of the pressure forces relative to the origin after simple computations

$$P = \gamma z_2(\lambda), \qquad M = \beta z_3(\lambda) \tag{3.6}$$

where

$$z_2(\lambda) = A^{-1}\,(2\lambda^{-1} + \sqrt{4/C} - B^{-1}) + (1 - \sqrt{C}/B)^2 \exp(-2B\lambda^{-1})$$

$$z_3(\lambda) = A^{-1}\,({}^2/{}_3\lambda + \sqrt{4/C} - B^{-1}) + 2\lambda b^2 + 2\lambda^2 \sqrt{A}\,[b^3 + (1 - \sqrt{C}/B)^2/\sqrt{B^3}]$$
$$- [1 + \lambda\,(B^{-1} - b\sqrt{A})]^2 \exp(-2B\lambda^{-1}), \quad b = (2B\sqrt{A})^{-1} - B^{-1/2} \tag{3.7}$$

Now, utilizing the statics condition (2, 7) and taking account of (3, 5)–(3, 7), we obtain an equation to determine λ

$$\lambda_1 = \frac{1}{\lambda}\left(1 + \frac{z_3(\lambda)}{z_1(\lambda)\, z_2(\lambda)}\right), \quad \lambda_1 = \frac{l_1 g}{U^2} \tag{3.8}$$

Let us note that, just as the analogous equation (2, 8) obtained for large λ, it is more convenient to solve (3, 8) by constructing graphs of the dependence of λ_1 on λ. Determining λ by the first formula in (3, 6), we find γ for a given P. Then by (3, 5) we find the slope of the plate as well as the drag $W = \beta P$.

4. The case of small λ is considered as before, but for $\delta > 1$. This latter circumstance influences obtaining the approximate solution substantially, since the Fourier transform of the kernel (3, 2) here has two symmetric poles on the real axis, and the kernel itself does not vanish at infinity ($t \to \infty$). The integral equations of the first kind with kernels of this type have already been investigated in [6], where the solution is given in complex form. An approximate solution based on the representation of the solution in the form (3, 1) is presented here.

Let us transform the integral equation (3, 2) into

$$\int_0^\infty p_j(u)\, K(t - u)\, du = \begin{cases} L_j(t), & t \geqslant 0, \ j = 1,2 \\ l_j(t), & t < 0, \ j = 1,2 \end{cases} \tag{4.1}$$

$$K(\alpha) = \int_0^\infty \left(\frac{1}{\tau \operatorname{cth} \delta\tau - 1} - \frac{2ac}{\tau^2 - c^2}\right) \cos \alpha\tau\, d\tau \tag{4.2}$$

$$L_1(t) = \pi\lambda^{-1} f_1(t) + 2\pi a \int_0^t p_1(u) \sin c\,(t - u)\, du \tag{4.3}$$

$$L_2(t) = \pi\lambda^{-1} f_2(t) - 2\pi a \int_t^\infty p_2(u) \sin c(t-u)\,du$$

where the integrand in (4.2) has no singularities on the real axis. It follows directly from (4.1)–(4.3) that the unknown functions $l_j(t)$ introduced tend to zero as $t \to -\infty$.

Applying the Fourier transformation with complex parameter α to (4.1), (4.2), we arrive at the following functional equations:

$$P_1^+(\alpha)\frac{k(\alpha)}{\alpha^2 - c^2} = F_1(\alpha) + E_1^-(\alpha) \tag{4.4}$$

$$P_2^+(\alpha)\frac{k(\alpha)}{\alpha^2 - c^2} = F_2(\alpha) + \frac{2a}{\alpha^2 - c^2}\left[i\alpha \operatorname{Im} P_2^+(c) + c \operatorname{Re} P_2^+(c)\right] + E_2^-(\alpha) \tag{4.5}$$

where

$$P_j^+(\alpha) = \frac{1}{\sqrt{2\pi}}\int_0^\infty p_j(t)\, e^{i\alpha t} dt, \qquad F_j(\alpha) = \frac{1}{\sqrt{2\pi}}\int_0^\infty f_j(t)\, e^{i\alpha t} dt$$

$$E_j^-(\alpha) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^0 l_j(t)\, e^{i\alpha t} dt, \qquad j = 1, 2, \qquad k(\alpha) = \frac{\alpha^2 - c^2}{\alpha \operatorname{cth} \delta\alpha - 1}$$

and $P^+_j(\alpha)$ are functions analytic in the upper half-plane $\operatorname{Im}\alpha > 0$ and decreasing as $\alpha^{-1/2}$ at infinity, and $E^-_j(\alpha)$ are functions analytic in the lower half-plane $\operatorname{Im}\alpha < \alpha_1$, $\alpha_1 > 0$. The function $k(\alpha)$ has no poles on the real axis. Let its factorization have the form

$$k(\alpha) = k^+(\alpha)\,[k^-(\alpha)]^{-1} \tag{4.6}$$

where $k^+(\alpha)$ and $k^-(\alpha)$ are analytic functions having no zeros in the half-planes $\operatorname{Im}\alpha > 0$ and $\operatorname{Im}\alpha < \alpha_1$, respectively. Let us note that $k^+(\alpha)$ grows as $\sqrt{\alpha}$ at infinity, while $k^-(\alpha)$ diminishes as $\alpha^{-1/2}$. Let us substitute (4.6) into (4.4), (4.5), and let us write the functional equations as

$$P_1^+(\alpha)k^+(\alpha) = (\alpha^2 - c^2)k^-(\alpha)F_1(\alpha) + (\alpha^2 - c^2)k^-(\alpha)E^-_1(\alpha) \tag{4.7}$$

$$P_2^+(\alpha)k^+(\alpha) = (\alpha^2 - c^2)k^-(\alpha)F_2(\alpha) + 2ak^-(\alpha)\left[i\alpha \operatorname{Im} P^+_2(c) + c \operatorname{Re} P^+_2(c)\right] + (\alpha^2 - c^2)k^-(\alpha)E^-_2(\alpha) \tag{4.8}$$

Let us make still another factorization

$$(\alpha^2 - c^2)k^-(\alpha)F_j(\alpha) = F_j^+(\alpha) + F_j^-(\alpha) \qquad (j = 1, 2)$$

where $F_j^+(\alpha)$ and $F_j^-(\alpha)$ are analytic functions in the half-planes $\operatorname{Im}\alpha > 0$ and $\operatorname{Im}\alpha < \alpha_1$, respectively. The functional equations may now be rewritten thus

$$P_1^+(\alpha)k^+(\alpha) - F_1^+(\alpha) = F^-_1(\alpha) + (\alpha^2 - c^2)k^-(\alpha)E^-_1(\alpha) \tag{4.9}$$

$$P_2^+(\alpha)k^+(\alpha) - F_2^+(\alpha) = F_2^-(\alpha) + (\alpha^2 - c^2)k^-(\alpha)E_2^-(\alpha) + 2ak^-(\alpha)\left[i\alpha \operatorname{Im} P_2^+(c) + c \operatorname{Re} P_2^+(c)\right] \tag{4.10}$$

The right and left sides of (4.9), (4.10) are analytic functions in the half-planes $\operatorname{Im}\alpha > 0$ and $\operatorname{Im}\alpha < \alpha_1$, respectively. Applying the theorem on analytic continuation, and the generalized Liouville theorem, and taking account of the behavior of $k^+(\alpha)$ and $P_j^+(\alpha)$ at infinity, we obtain that the functions identically equal the constants C_1 and C_2. Furthermore, we have

$$P_j^+(\alpha) = \frac{C_j + F_j^+(\alpha)}{k^+(\alpha)} \qquad (j = 1, 2), \qquad E_1^-(\alpha) = \frac{C_1 - F_1^-(\alpha)}{(\alpha^2 - c^2)k^-(\alpha)}$$

$$E_2^-(\alpha) = \frac{C_2 - F_2^-(\alpha) - 2ak^-(\alpha)\left[i\alpha \operatorname{Im} P_2^+(c) + c \operatorname{Re} P_2^+(c)\right]}{(\alpha^2 - c^2)k^-(\alpha)} \tag{4.11}$$

On the basis of the above regarding the behavior of the function $l_j(t)$ as $t \to -\infty$ we conclude that the functions $E_j^-(t)$ should have no poles on the real axis. This results in the conditions

$$C_1 - F_1^-(\pm c) = 0, \quad C_2 - F_2^-(\pm c) - 2ack^-(\pm c)\, P_2^+(\pm c) = 0 \qquad (4.12)$$

The second of conditions (4.12) is satisfied identically by virtue of (4.11), which permits setting $C_2 = 0$ (the boundedness of the pressure at the point of convergence $x = 1$). The first condition of (4.12) defines C_1 and, besides, imposes specific requirement on the function $f(x)$.

In the case under consideration, assigning the functions $f_j(t)$ by the expressions (3.3), we have

$$P_j^+(\alpha) = \frac{\lambda i\alpha^2 C_j - c^2[(-1)^j(\lambda i - \alpha)\beta k^-(0) - i\gamma\alpha k^-(0) + \lambda(-1)^j[k^-(0)'i\alpha]}{\sqrt{2\pi}\, k^+(\alpha)\,\alpha^2 i\lambda} \qquad (4.13)$$

where

$$C_1 = -\beta k^-(0), \qquad C_2 = 0, \qquad (\gamma - \beta)\, k^-(0) + i\beta\,[k^-(0)]'\,\lambda = 0 \qquad (4.14)$$

The last condition in (4.14) is indeed that requirement which the function $f(x)$ should satisfy; it determines the connection between γ and β.

The solutions $p_j(t)$ of the integral equations (3.2) are determined from (4.13) by a Laplace inversion. We shall hence utilize the approximation

$$k(\alpha) = \frac{\alpha^2 - c^2}{\alpha \operatorname{cth}\delta\alpha - 1} = \frac{\alpha^2 + D^2}{\alpha^2 + C^2}\sqrt{\alpha^2 + B^2}, \qquad A = \frac{D^2 B}{C^2} = \frac{\delta c^2}{\delta - 1} > 0$$

which results in the following expressions:

$$p_j(t) = C_j\left[\frac{e^{-Bt}}{\sqrt{\pi t}} + \frac{C - D}{\sqrt{B - D}}\, e^{-Dt} \operatorname{erf}\sqrt{(B - D)t}\right] + \qquad (4.15)$$

$$+ \frac{c^2}{A}\left[(-1)^j\beta\varepsilon + \frac{\gamma + (-1)^j\beta}{\lambda}\right]\left[\frac{(C - D)\sqrt{A}}{D\sqrt{B - D}}\, e^{-Dt}\operatorname{erf}\sqrt{(B - D)t} - \operatorname{erf}\sqrt{Bt}\right] +$$

$$+ (-1)^j\frac{\beta c^2}{A}\left[(t + \varepsilon)\operatorname{erf}\sqrt{Bt} + \sqrt{\frac{t}{\pi B}}\, e^{-Bt} + \frac{(C - D)\sqrt{A}}{D^2\sqrt{(B - D)}}\, e^{-Dt}\operatorname{erf}\sqrt{(B - D)t}\right]$$

$$\varepsilon = \frac{1}{C} - \frac{1}{D} - \frac{1}{2B}, \quad C_1 = -\frac{\beta}{\sqrt{A}}, \quad \gamma = \beta(1 + \varepsilon\lambda), \quad C_2 = 0$$

It is not difficult to obtain the degenerate solution from (4.15) for $\lambda \to 0$. We have

$$q(x) = -c^2(A\lambda)^{-1}(\gamma + \beta x) \qquad (4.16)$$

In combination with (3.1), formulas (4.15), (4.16) determine the desired pressure $p(x)$. Let us find the lift P, and the moment M relative to the origin acting on the gliding plate

$$P = -\beta\varphi_1(\lambda), \quad M = \beta\varphi_2(\lambda) \qquad (4.17)$$

$$\varphi_1(\lambda) = \lambda A^{-1}[\varkappa_1(\lambda) - \varkappa_2(\lambda)] + 2c^2(A\lambda)^{-1}(1 + \varepsilon\lambda)\,[-1 + (2 + \lambda\varepsilon)\varkappa_1(\lambda) +$$
$$+ \sqrt{2\lambda/\pi B}\exp(-2B\lambda^{-1}) + \lambda D^{-1}\varkappa_2(\lambda)] \qquad (4.18)$$

$$\varphi_2(\lambda) = -2c^2A^{-1}\{\varkappa_1(\lambda)\,[{}^2/_3\lambda + \varepsilon + \lambda(\varepsilon^2 - {}^1/_2c^2) + \lambda^2(\sigma_2 - \varepsilon/2c^2)] -$$
$$- {}^1/_3\lambda - D^{-1}\varkappa_2(\lambda)\,[1 + \lambda(\varepsilon + 2D^{-1} + D/2c^2) + \lambda^2(\varepsilon D^{-1} + D^{-2} + {}^1/_2c^2)] +$$
$$+ \sqrt{2(\lambda\pi B)^{-1}}\exp(-2B\lambda^{-1})\,[{}^1/_3 + \lambda(\varepsilon - {}^1/_3B) + \lambda^2(\sigma_1 - {}^1/_2c^2)]\}, \qquad \varkappa_1 = \operatorname{erf}\sqrt{2B\lambda^{-1}}$$

$$\varkappa_2 = \frac{(C - D)\sqrt{A}}{D\sqrt{B - D}}\, e^{-\frac{2D}{\lambda}} \operatorname{erf}\sqrt{\frac{2(B - D)}{\lambda}}, \qquad \sigma_1 = \varepsilon^2 + \frac{\varepsilon}{2B} - \frac{C - D}{CD^2} - \frac{1}{4B^2}$$

$$\sigma_2 = (C - D)\, D^{-2}C^{-2} - \varepsilon^2(2B)^{-1} - \varepsilon(4B^2)^{-1} + (8B^3)^{-1}$$

Analogously to Sect. 2, the equilibrium condition for the plate must be added to the relationships (4.17), (4.18); then we obtain for the determination of the quantity λ

$$\lambda_1 = l_1 g U^{-2} = \lambda^{-1} [1 + \varphi_2 (\lambda) / \varphi_1 (\lambda)] \tag{4.19}$$

Let us find the drag experienced by the plate

$$W = - P^2 / \varphi_1 (\lambda)$$

The case $\delta = 1$ cannot be considered on the basis of linear theory.

As an illustration, let us present some results of calculations for the cases $\delta = 0.5$ and $\delta = 2$. It turns out that the value of the moment relative to the origin and the lift coefficient obtained by both methods for $\delta = 0.5$ differ by 6% and 2% for $\lambda = 3.5$, respectively, and by 4% and 3%, respectively, for $\delta = 2$ and $\lambda = 2.5$.

Let us note that the distance of the center of pressure from the trailing edge of the plate tends to $^4/_3 l$ for $\lambda \to 0$ in the case $\delta < 1$ and to $^2/_3 l$ as $\lambda \to 0$ for the case $\delta > 1$.

Let us note that although the authors have not yet succeeded in providing a rigorous foundation for the method proposed in Sect. 4, as has been done, say, in the fundamental work of Babeshko [6], the numerical results show that the zero term of the asymptotics obtained for small λ in Sect. 4 is sufficiently effective.

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